OPTIMIZATION OF HYPERSONIC THREE-DIMENSIONAL SHAPES

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The construction of optimal hypersonic aerodynamic shapes is an urgent problem. In the class of complex three-dimensional configurations, this construction is still realized on the basis of approximate methods of analyzing the hypersonic flow (see [1-4], for instance).

Within the framework of a Newtonian flow scheme with a friction correction, a new class of three-dimensional configurations with power-law longitudinal and transverse contours, including particularly bodies of revolution, multicantilever wings, a curvilinear multiwedge body with a circular middle, is considered below. The values of the parameters describing the optimal streamlined surface are determined by the method of random search by means of the best sample [5] from the condition of minimum total body drag. It is established as a result of computations that the optimal hypersonic shapes in the span range $0<\lambda<4$ are three-dimensional bodies with starlike middles.

Experimental values of the total drag coefficient are presented for certain configurations which agree qualitatively with the computed data.

1. Three-dimensional bodies of unit length whose equation of the cyclic surface (Fig. 1) is represented in the form [1]

$$f \equiv r - \rho x^{k_1}\left[1 + k_2(1-k_3x)^{k_4}\, e^{-k_5x}\left(\theta\,\frac{n}{\pi}\right)^{\frac{k_6 e^{k_5 x}}{(1-k_3x)k_4}}\right] = 0, \tag{1.1}$$

where r, θ, x are components of a cylindrical coordinate system, ρ is the ratio between the radius of the base (for $x=1$, $k_2=0$) circumference of the body base and its length, n is the quantity of transverse body cycles, and k_i ($i=1, \ldots, 6$) are the "inner" variable parameters (ρ, n are the "outer" (given) parameters).

The independent variables are enclosed in the ranges $\varepsilon \le x \le 1$, $\varepsilon \le (\theta n/\pi) \le 1$, where ε is a small quantity selected from the condition constraining the appropriate partial derivatives of the function f. The variable parameters vary between the limits

$$\begin{gathered} 0{,}1 \leqslant k_1 \leqslant 5,\ 0 \leqslant k_2 \leqslant 10,\ 0 \leqslant k_3 \leqslant 1,\ 0 \leqslant k_4 \leqslant 5, \\ 0 \leqslant k_5 \leqslant 10,\ 0 \leqslant k_6 \leqslant 50;\ 0 \leqslant n \leqslant 100,\ 0{,}025 \leqslant \rho \leqslant 10. \end{gathered} \tag{1.2}$$

The ranges of variation of the "inner" variable parameters are chosen so that the extremal point will always be an inner point as the "outer" parameters vary.

The function f (1.1) represented describes a class of streamlined surfaces without "aerodynamic shadow" under the conditions $r_x\big|_{\theta=\frac{\pi}{n}} \geqslant 0, r_x|_{\theta=0} \geqslant 0$. A lower bound on the radius of curvature along the inner normal of the running transverse contour is introduced for a slight twist in short and medium bodies. The lower bound of the so-called radius of curvature equals the transverse coordinate of the inner edge (ONB in Fig. 1) in the same section.

Particular kinds of the class of aerodynamic shapes to be studied are the cone ($k_1=1$, $k_2=0$), the power-law body of revolution ($n=0$ or $k_2=0$), the body of revolution with radial plates ($k_6\to\infty$) and slits ($k_6\to 0$), a starlike body with power-law longitudinal contour from [1] ($k_1=0.75$, $k_6\approx 2$, $k_3=k_4=k_5=0$), a starlike body from [6] ($k_2\approx 0.2$, $0<k_5<0.05$, $k_6\approx 5$, $k_1=1$, $k_4=0$), the nose section with power-law longitudinal and transverse contours with a circular base ($k_3=1$, $k_4\ge 1$). Different V-wing shapes are obtained upon gluing just two cyclic elements, and a wing with a fuselage ($n=2$) is designed diversely.

2. A modified Newtonian model of the flow with a variable surface friction coefficient C_f

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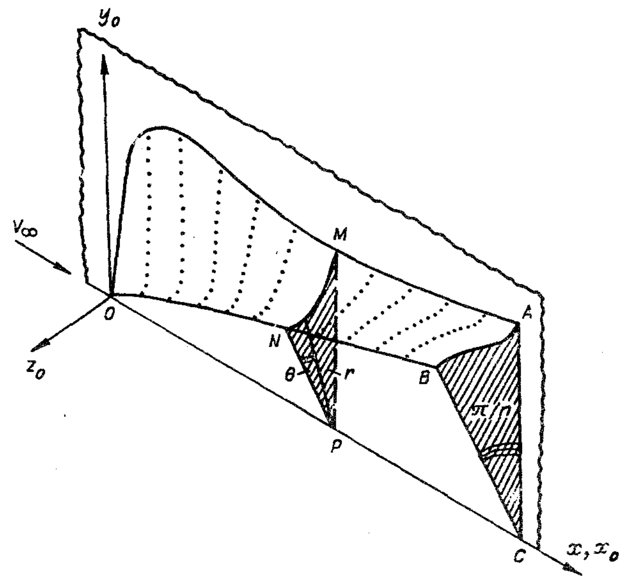

Fig. 1

$$C_p = 2.091(\mathbf{l}_n\mathbf{l}_x)^2, \; C_f = C_{f0}(1-\eta)x^{-\eta},$$

is used to describe the aerodynamic drag of the configurations being studied, where C_p is the pressure coefficient, $\mathbf{l}_n$ is the unit vector normal to the surface, $\mathbf{l}_x$ is the unit vector of the Ox axis, C_{f0} is the mean value of the friction coefficient with respect to the length, usually taken equal to $3 \cdot 10^{-3}$ for hypersonic investigations, and η is a constant equal to 0.5 for a laminar stream. Then the expression for the drag X referred to the velocity head q is written as

$$\frac{X}{q} = \int_0^{\pi/n}\int_0^1 \frac{r}{f_r}\left[-2.091 f_x^3\left(f_r^2 + \frac{f_\theta^2}{r^3} + f_x^2\right)^{-1} + C_f\left(f_r^2 + \frac{f_\theta^2}{r^2}\right)^{0,5}\right] d\theta dx, \tag{2.1}$$

where f_x, f_r, f_θ are partial derivatives of the function f with respect to the corresponding variables.

In the numerical integration of (2.1), the wave drag of sections of the surface (1.1), which are in the "aerodynamic shadow" on the downwind side of a possible wedge projection, vanishes. The drag of projections which are manifest upon variation in the "aerodynamic shadow" itself is considered by means of the unperturbed scheme.

Consequently, for given spans λ (or $\rho = {}^1\!/_2\lambda$) and a quantity n of transverse cycles, the problem is formulated as follows. The drag integral (2.1) is minimized in a set of parameters k_i defined by the relations (1.2), upon compliance with the constraint on the transverse contour for bodies with the span $\lambda < 4$. The optimization is performed by the method of random search on the best sample.

3. It is found because of the optimization that three-dimensional bodies with starlike cross section including the middle are best from drag respects for spans $\lambda < 4$ in the hypersonic range. Thus, it is seen from Fig. 2a, where configurations of the outer and inner (solid and dashed lines, respectively) edges $r_*(x)$ of optimal bodies with four transverse cycles, that three-dimensional aerodynamic shapes with a circular middle are the transition configurations from the power-law bodies of revolution to bodies with a starlike base. Axisymmetric bodies with an increasing exponent for the longitudinal generator as the span increases (Fig. 2b) are optimal for large spans. For $\lambda > 10$ the optimal axisymmetric bodies are simulated by aerodynamic shapes with a nose spike. The edge configurations shown in Fig. 2c are obtained upon removing the constraint on the transverse contour for small and medium spans. The boundary values r_* of the radius vector r corresponding to the angles $\theta = \pi/n$ (outer edge) and $\theta = 0$ (inner edge) are plotted along the ordinary axis in Fig. 2. The edges of bodies of the same span are connected by line segments collected from points for $x = 1$.

It is seen from a comparison of the transverse contours of four-wedge bodies $r(\theta)|_{x=\mathrm{const}}$ (Fig. 3a-d are the results of optimization with a constraint on the radius of curvature, and Fig. 3e-h are the results of free optimization) that for small spans the constraint on the cross section of the body being optimized substantially weakens the rotation of the transverse generators with respect to each other, thereby practically eliminating the stream twist. It is also shown in Fig. 3 how the transverse body contours vary with the change in span. Fig. 3a and e correspond to the body span $\lambda = 0.125$, Fig. 3b and f to $\lambda = 0.75$, Fig. 3c and g to $\lambda = 2.25$, and Fig. 3d and h to $\lambda = 4$.

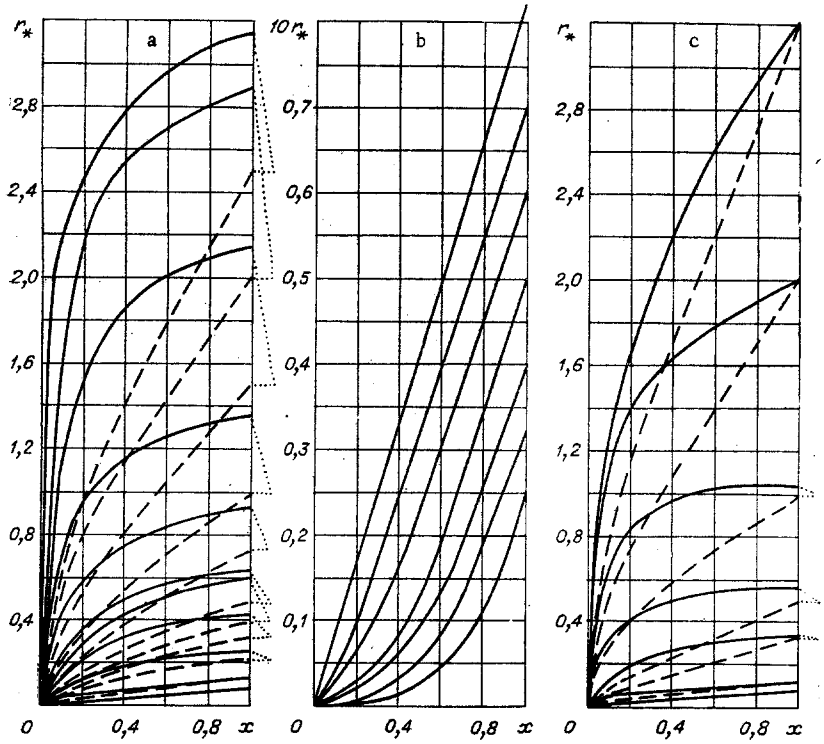

Fig. 2

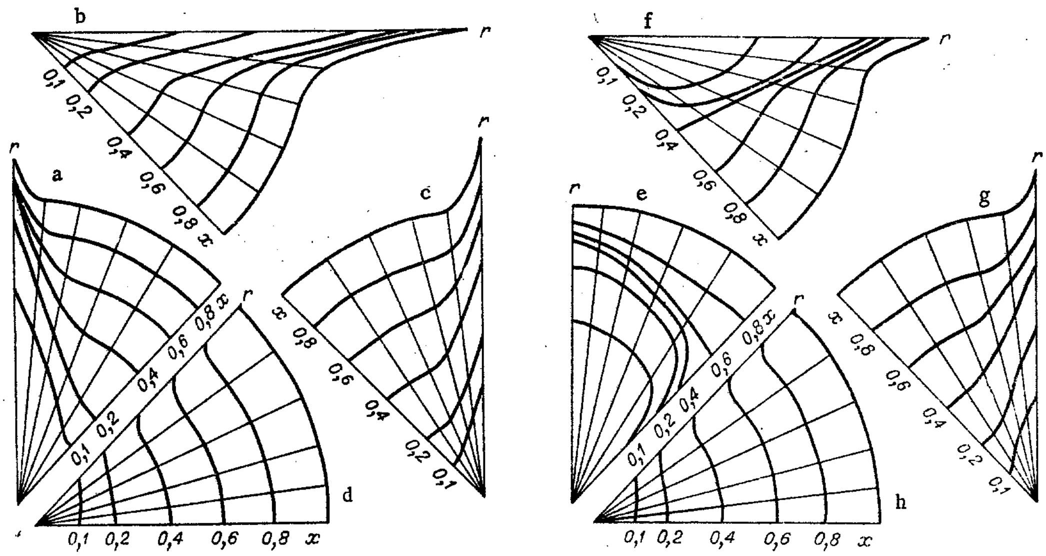

Fig. 3

4. It should be noted that four-wedge configurations with the parameters $k_1 = 0.764$, $k_2 = 2.462$, $k_3 = 0.027$, $k_4 = 1.999$, $k_5 = 1.463$, and $k_6 = 2.346$ in the case of optimization with a constraint, and $k_1 = 0.827$, $k_2 = 4.412$, $k_3 = 0.330$, $k_4 = 1.053$, $k_5 = 2.878$, and $k_6 = 0.422$ in the case of free optimization, have the maximum emergence of the outer edges beyond the diameter of the base circumference of the base for the span $\lambda = 0.75$ (see Fig. 2a and c), and reach the maximum reduction in drag relative to the equivalent cone. The solid line in Fig. 4 shows the dependence of the ratio between the drag X_c of a cone or equivalent span and the drag X_{ob} of the optimally constructed four-wedge body with a constraint on the transverse contour, while the dashed line is for a four-wedge body obtained during free optimization, and the dash–dot line is an analogous dependence for a power-law axisymmetric minimum-drag body. It is seen from Figs. 2 and 4 that aerodynamic shapes with a starlike cross-section have least drag for small and medium spans, while power-law bodies of revolution assure minimum drag for large spans.

5. The influence of the quantity of transverse cycles n on the drag of three-dimensional bodies optimized with a constraint on the transverse contour, is shown in Fig. 5 for three spans (curve 1 corresponds to $\lambda = 0.75$;

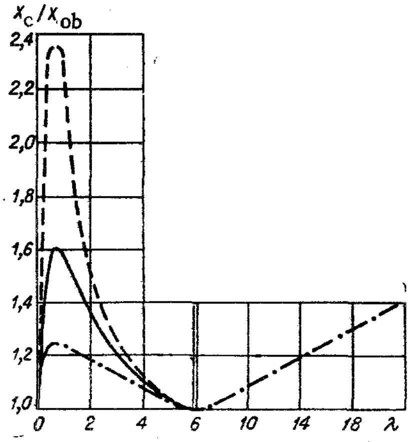

Fig. 4

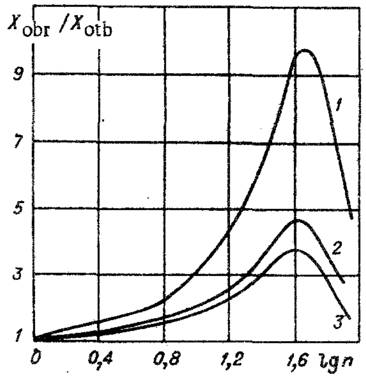

Fig. 5

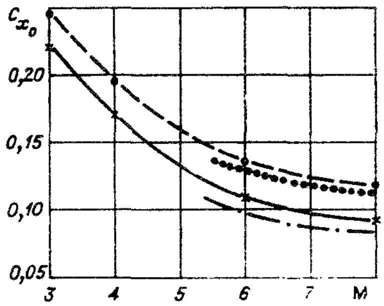

Fig. 6

2 to $\lambda = 0.125$; 3 to $\lambda = 2.25$) where the ratio between the drag of an optimal body of revolution X_{obr} and the drag of an optimal three-dimensional body X_{otb} is plotted along the ordinate axis. It is seen in Fig. 5 that the extremum point for the spans considered is near $n = 40$ ($\log n = 1.6$), where $\lambda = 0.75$ at the "peak" span, and the ordinate of the maximum is greatest here. Taking account of the interference of shock layers at the junctures of the cyclic surfaces [7] permits refinement of the optimal quantity of transverse cycles for plane and ruled aerodynamic shapes with sharp leading edges [8, 9]. Taking account of the secondary collision of the gas particles on the surface in the inner edge region is fraught with great computational difficulties for curvilinear three-dimensional bodies [10] since the parameter distribution within the shock layer must be obtained for this. Because of the absence of the mentioned accounting and estimates of the forces concentrated on the real leading edges, the dependences $X_{obr}/X_{otb} = f(n)$ obtained for large n require refinement.

6. A comparison of the computations by means of a modified Newtonian approximation to describe the drag of three-dimensional bodies with a small quantity of transverse cycles and the experimental dependences of the total drag coefficient C_{x_0} at zero angle of attack on the Mach number M for an equivalent cone (dashed line) and a four-wedge body with the parameters $k_1 = 0.742$, $k_2 = 0.724$, $k_3 = 1.0$, $k_4 = 2.491$, $k_5 = 2.538$, and $k_6 = 6.915$ (solid line) is shown in Fig. 6 for the span $\lambda = 2.25$. The tests were performed in the T-313 and IT-301 wind tunnels of the ITPM SO AN SSSR (Institute of Theoretical and Applied Mechanics, Siberian Branch of the USSR Academy of Sciences) for M = 3, 4, 6, 8 and the Reynolds number $Re \approx 10^7$ on models with the middle diameter 60 mm and the leading edge thickness 0.2 mm. The points indicate the computational dependence $C_{x_0}(M)$ for the cone, and the dash–dot line is for the three-dimensional body. Both computational dependences are below their corresponding experimental graphs, where the discrepancy between the results of the computation and the experiment diminishes as the Mach number increases. Despite the somewhat more degraded agreement between the computed and experimental data, as compared to the cone, the accuracy obtained in the description C_{x_0} of the experimental four-wedge body permits the hope for qualitatively correct results of the optimization of three-dimensional aerodynamic shapes with a low number of cross-section cycles and a constraint on the contour and for smaller spans, where the maximum computed drag reductions are observed.

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CROSSFLOW DEVELOPMENT IN THE BOUNDARY LAYER DURING LONGITUDINAL FLOW AROUND A RIGHT DIHEDRAL ANGLE

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Cases of the interaction of both laminar and turbulent boundary layers are realized in the flow around angular configurations. Investigations [1, 2] executed earlier indicate that during the interaction of turbulent boundary layers in the neighborhood of a bisectorial plane of a corner, crossflows in the form of counter-rotating vortex pairs develop.

This paper is devoted to an experimental investigation of the conditions for the origination and development of crossflows in the domain of boundary-layer interaction during the transition from the laminar to the turbulent state.

The tests were performed in the low-turbulence T-324 wind tunnel of the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the USSR Academy of Sciences [3] under conditions of gradient-free flow around a right dihedral angle model. The description of the model construction and the fundamental measurement methodology are elucidated in [2]. The experiments were conducted at a mean stream velocity of $u_\infty = 7.6$ m/sec and a Reynolds number $Re_1 \simeq 0.53 \cdot 10^6$, m^{-1}, initial degree of turbulence $\varepsilon \simeq 0.03\%$, and zero radius of conjugation of the angle faces in the working section. A constant temperature thermoanemometer 55D00 of the firm DISA in connection with a 55D10 linearizer was used as recording apparatus in measuring the longitudinal velocity component and its pulsations. A transducer with 0.65-mm-long and 3-μm-diameter Wollaston wire was used in the majority of tests, which assured a sufficiently low time constant τ. The working frequency band hence exceeded 40 kHz. All this permitted obtaining an acceptable resolution of the thermoanemometer system as a whole under the investigated conditions. The spectral characteristics of the velocity pulsations were investigated by using a frequency analyzer of 2010 type of the firm of Brüel and Kjer. The experience accumulated in the Institute of Theoretical and Applied Mechanics and other organizations [4-9], was used in measuring the velocity pulsations with the thermoanemometer.

The results of investigating the laminar-into-turbulent boundary-layer transitions during the flow around a right dihedral angle at supersonic speeds [10] showed that the boundary layer in the bisectorial plane becomes turbulent directly from the leading edge although laminar boundary layers are realized and interact on a certain section outside the domain of interaction on the faces of the angle.

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